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Irregular Spatial Sampling and Rank-reduction - Interpolation by Joint Low-rank and Sparse Inversion

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SUMMARY

Until now noise attenuation and interpolation processes based on rank reduction needed spatially regular, or at least binned, data. Joint low-rank and sparse inversion (JLRSI) has been recently proposed as a convex optimization framework for simultaneous random plus erratic noise attenuation and interpolation. We show how the low-rank signal model in JLRSI can be extended to spatially irregular data by appropriately modifying the inverse problem formulation. Benefits of considering the true spatial locations of seismic traces for the quality of the signal reconstruction are illustrated on a three-dimensional regularization and interpolation example on real land data.

Introduction

Rank-reduction methods such as Cadzow/singular spectrum analysis (SSA) filtering (Trickett 2008, Sacchi 2009) have been extended to robust noise attenuation to deal with erratic noise, or data interpolation for binned data within a defined grid (Trickett et al. 2010, 2012, Oropeza and Sacchi 2011, Chen and Sacchi 2013). Sternfels et al. (2015) recently proposed the joint low-rank and sparse inversion (JLRSI) convex optimization framework to address the simultaneous random plus erratic noise attenuation and interpolation problem. In this approach the laterally coherent signal was modelled via a low-rank trajectory matrix in the spirit of Cadzow/SSA filtering, while the spatially localized erratic noise was a sparse component of the input data.

However, these rank-reduction techniques work best when the assumption of spatially regular data is satisfied. When dealing with highly irregular datasets, they do not have the advantage of methods based on irregular transforms such as the anti-leakage Fourier transform (Xu et al. 2005, 2010, Poole 2010) or non-equispaced sparsity-based transforms (Hennenfent et al. 2010, Wang et Nimsaila 2014), which honour exact trace coordinates and avoid binning, thus improving the reconstruction accuracy.

In this work we generalize the JLRSI strategy to natively handle irregular spatial sampling. This is achieved by a modification of the inverse problem formulation of JLRSI through the use of a regular-to-irregular mapping honouring the exact trace coordinates. The ability to deal with highly irregular seismic data enables effective use of the proposed extended JLRSI framework for robust data regularization and interpolation, as demonstrated in the real data example.

Theory

We start by giving a quick review of the Cadzow/SSA signal model and its use in the JLRSI framework. For a given spatiotemporal analysis window, we work in the temporal frequency domain and model the laterally coherent signal at each frequency by a low-rank trajectory matrix. For one spatial dimension (e.g., inline direction) and a spatial window of five traces, the trajectory matrix for a given frequency slice of the desired signal $S = (S_1,\ldots,S_5)$ has the form of a low-rank Hankel matrix:

$$H(S) = \begin{pmatrix} S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \\ S_3 & S_4 & S_5 \end{pmatrix}.$$

Cadzow/SSA filtering recovers the signal component S by a direct rank reduction of the trajectory matrix $H(D)$ formed from a frequency slice of the input data $D = (D_1,\ldots,D_5)$. This is achieved by performing a singular value decomposition (SVD) of $H(D)$ and discarding the smaller singular values corresponding to random noise. We point out that the signal model can be generalized for higher spatial dimensions by organizing the frequency slice data in a multi-level Hankel matrix with Hankel blocks (Trickett 2008, Oropeza and Sacchi 2011).

JLRSI allows for the recorded data D to be both incomplete and corrupted by erratic noise such as strong localized spike bursts or phase distortions. Applying Cadzow/SSA filtering directly to such data would yield incorrect results because the SVD is optimal only in the least-squares sense, thus very sensitive to missing data and outliers. To address this problem, JLRSI introduces a sparse erratic noise term E and a sampling operator $P_\Omega[.]$ (zero when data are unavailable, identity otherwise), and formulates the simultaneous random plus erratic noise attenuation and interpolation problem as a constrained convex optimization problem:

$$\begin{aligned} &\text{minimize} && \| H(S) \|_* + \lambda \| E \|_1 , \\ &\text{such that} && \| P_\Omega[D - S - E] \|_2 \leq \delta. \end{aligned}$$

The nuclear norm (sum of singular values) $\|H(S)\|_*$ of the trajectory matrix acts as a rank penalty, and the L_1 norm $\|E\|_1$ of the erratic noise component emphasizes sparsity. The constraint enforces the low-rank and sparse components S and E to fit the available data D up to an assumed level δ of random noise. The JLRSI convex optimization problem is solved using the alternating direction method of multipliers (ADMM) algorithm (Parikh and Boyd 2013); details of the implementation are provided in Sternfels et al. (2015).

For spatially irregular data, one can use binning in order to be able to map the input frequency slice data to trajectory matrix form. However, as illustrated in the examples section, using the exact trace coordinates allows for a better signal reconstruction; this is exemplified by the fact that methods such as the anti-leakage Fourier transform (Xu et al. 2005, 2010, Poole 2010) are now a standard in the industry for regularization and interpolation of seismic data. To that aim, similarly to Kumar et al. (2015), we introduce a regular-to-irregular mapping operator $\boldsymbol{A}_{\mathrm{G}}$, which maps the regularized and interpolated signal S_{reg} back to the original trace coordinates. In practice, the operator $\boldsymbol{A}_{\mathrm{G}}$ can be implemented by a forward fast Fourier transform (FFT), mapping from a regular spatial grid to a regular wavenumber grid, followed by an adjoint unequally spaced FFT (Dutt and Rokhlin 1993, Duijndam and Schonewille 1999), mapping from a regular wavenumber grid to an irregular spatial grid. We point out that the simple application of the adjoint of $\boldsymbol{A}_{\mathrm{G}}$ for regularization purposes does not solve the problem because of spectral leakage, as demonstrated in Xu et al. (2005, 2010). The JLRSI formulation modified for spatially irregular data may be given by:

$$\begin{aligned} &\text{minimize} \quad && \| H(S_{\mathrm{reg}}) \|_* + \lambda \| E \|_1 , \\ &\text{such that} \quad && \| D - \boldsymbol{A}_{\mathrm{G}} S_{\mathrm{reg}} - E \|_2 \leq \delta. \end{aligned}$$

There is no sampling operator as the constraints are now directly formulated in the irregularly sampled domain. Compared to the original JLRSI problem, the frequency slices D and E corresponding to the input data and the erratic noise component are in the irregularly sampled domain, and the main addition resides in the regular-to-irregular mapping operator $\boldsymbol{A}_{\mathrm{G}}$. The extended JLRSI formulation is also solved using the ADMM algorithm; however one difficulty is introduced by the addition of the regular-to-irregular mapping operator $\boldsymbol{A}_{\mathrm{G}}$ in the S_{reg} update step within the ADMM iterations, which has the following form:

$$\operatorname{argmin}_{S\mathrm{reg}} \quad \tau \| H(S_{\mathrm{reg}}) \|_* + \tfrac{1}{2} \| \boldsymbol{A}_{\mathrm{G}} S_{\mathrm{reg}} - Z \|^2,$$

where τ and Z are respectively a scalar and frequency slice data appearing in the ADMM iterations. If $\boldsymbol{A}_{\mathrm{G}}$ were the identity operator, the solution of this sub-problem would be given by singular value soft-thresholding (Cai et al. 2010) as in the original JLRSI formulation. In this work, we use a single linearized proximal forward-backward splitting step, in the spirit of Combettes and Wajs (2005) or Osher et al. (2010), to approximately solve the S_{reg} sub-problem within the ADMM iterations. This is usually enough to observe convergence of the algorithm, with the only added expense being one additional evaluation of $\boldsymbol{A}_{\mathrm{G}}$ and its adjoint for each S_{reg} update.

Example

We applied the proposed modified JLRSI framework on a three-dimensional land dataset consisting of a cross-spread gather, with nominal shot and receiver spacing of respectively 100 and 50 meters. Because of the coarse spacing between shots and receivers, the lower frequency part of the ground roll is strongly aliased. Our motivation was to interpolate the cross-spread gather onto a regular, denser spatial grid with half nominal shot and receiver spacing; so that the ground roll would be partially or fully de-aliased to enable effective attenuation of the latter. We used two spatial dimensions in this example: inline and crossline, for which respective increments were half the shot and receiver spacing. Figure 1 shows the cross-spread irregular configuration, Figure 2 a shot gather extracted from the original data. The dispersive nature of surface waves required us to process each frequency slice independently; the aliasing ambiguity introduced by the regular-to-irregular mapping operator – when the regular grid is denser than the survey grid– is dealt with by initializing the process with a fast nearest-neighbour interpolation. The result from the proposed JLRSI reconstruction scheme is shown on Figure 3 for the same shot gather with conservative noise attenuation settings. For comparison purposes, on Figure 4 we show the result from the same reconstruction when using binned data and bin-centered coordinates. On the zoomed section we clearly see discrepancies between binned survey traces and interpolated traces due to the coarse spatial sampling and the irregular distribution of traces within each bin, in contrast to the proposed modified JLRSI formulation which provided more continuous and consistent events.

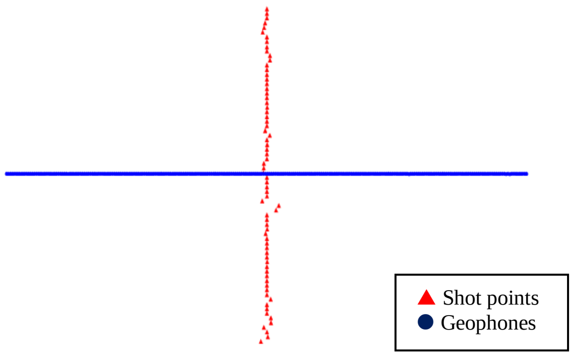

Figure 1 *Cross-spread configuration, shot point locations are noticeably irregularly spaced.*

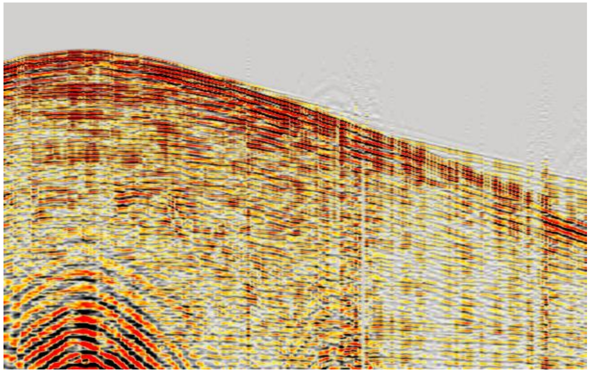

Figure 2 *Shot gather extracted from the original data after limited pre-processing.*

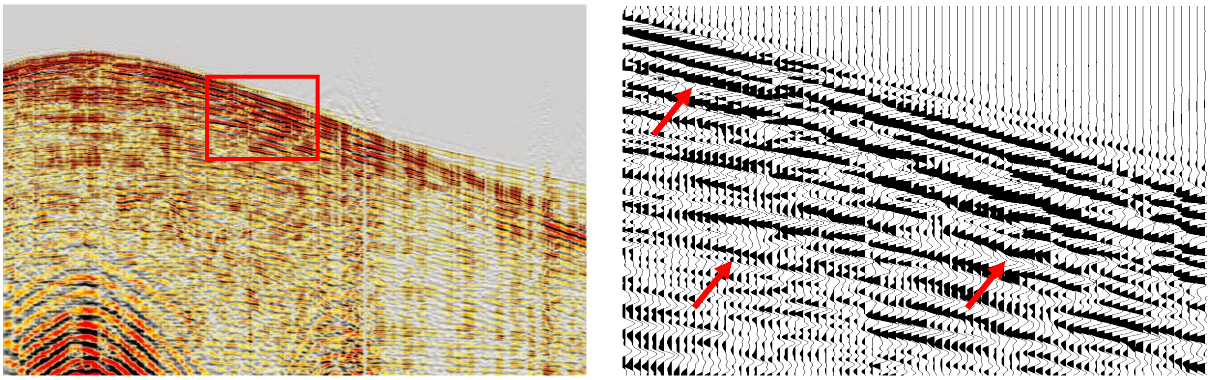

Figure 3 *Interpolated shot gather using true traces coordinates, zoomed rectangle on the right.*

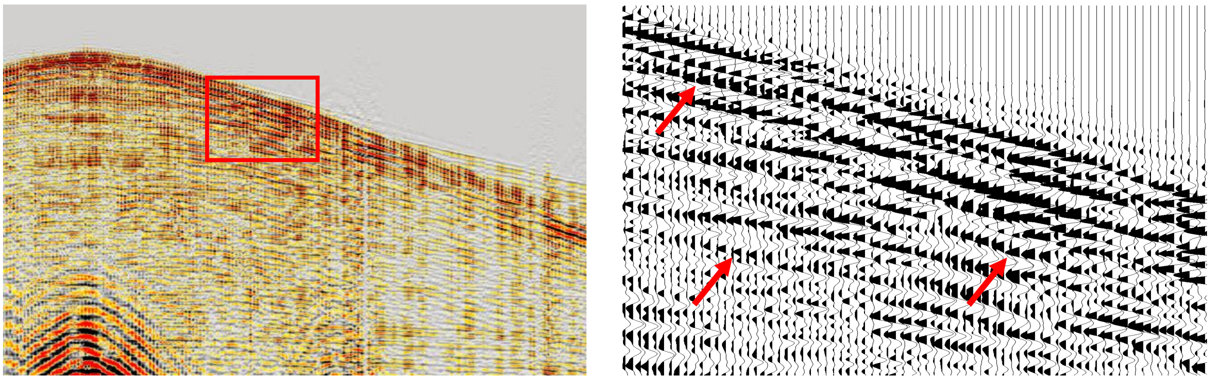

Figure 4 *Interpolated shot gather using bin-centered coordinates. Events continuity is inferior.*

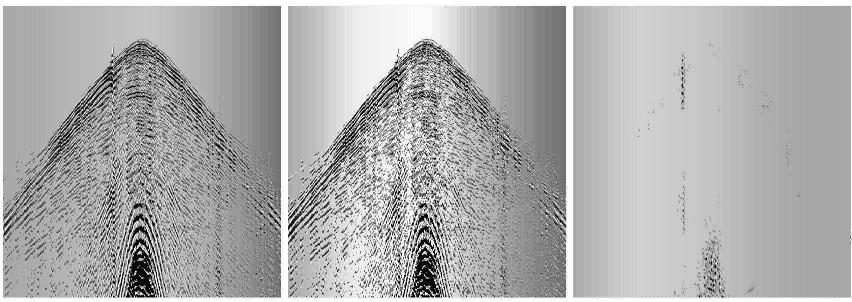

Figure 5 *Original shot gather (left), regularized shot gather after mapping back to original coordinates (middle), difference (right).*

On top of the irregular spatial sampling, the low amount of processing applied (before any surface-consistent processing such as amplitude, deconvolution, and refraction/residual statics) makes the reconstruction problem challenging, as lateral coherence for higher frequencies becomes limited. Figure 5 shows the difference between the original data and the regularized data from Figure 3 after mapping back with the regular-to-irregular operator; we see that the proposed JLRSI scheme for spatially irregular data provides minimal leakage of reflection signal, which can be further reduced for higher frequencies by applying static corrections beforehand.

Conclusion

We generalize the joint low-rank and sparse inversion (JLRSI) strategy for simultaneous denoising and interpolation to spatially irregular data by introducing a regular-to-irregular mapping operator in the inverse problem formulation. Simultaneous regularization/interpolation, erratic and random noise attenuation of spatially irregular multidimensional seismic data can then be performed by solving the resulting convex optimization problem via a modified ADMM algorithm. Finally, the effectiveness and accuracy of the presented method were illustrated on our real data example.

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